# Quantum Immortality and Non-Classical Logic

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**Abstract**

The *Everett Box* is a device in which an observer and a lethal quantum apparatus are isolated from the rest of the universe. On a regular basis, successive *trials* occur, in each of which an automatic measurement of a quantum superposition inside the apparatus either causes instant death or does nothing to the observer. From the observer's perspective, the chances of surviving $m$ trials monotonically decreases with increasing $m$. As a result, if the observer is still alive for sufficiently large $m$ she must reject any interpretation of quantum mechanics which is not the many-worlds interpretation (MWI), since surviving $m$ trials becomes vanishingly unlikely in a single world, whereas a version of the observer will necessarily survive in the branching MWI universe. Here we ask whether this conclusion still holds if rather than a classical understanding of limits built on classical logic we instead require our physics to satisfy a computability requirement by investigating the Everett Box in a model of a computational universe using a variety of constructive logic, Recursive Constructive Mathematics. We show that although the standard Everett argument rejecting non-MWI interpretations is no longer valid, we nevertheless can argue that Everett's conclusion still holds within a computable universe. Thus we claim that since Everett's argument holds not only in classical logic (with embedded notions of continuity and infinity) but also in a computable logic, any counter-argument claiming to refute it must be strengthened.

## 1 Introduction

The several interpretations of quantum mechanics can be divided into two classes. One class contains a single interpretation, the *many-worlds interpretation* (MWI) of Hugh Everett [Eve57b, Eve57a], so named by Bryce DeWitt [DeW70, DeW72]. In the MWI, the wavefunction never collapses and essentially the universe can be taken to be governed by a single, objectively real, universal wave function. Although when he introduced this idea in his thesis of 1957, Everett never spoke of a *branching* universe in which the universe splits into $n$ branches (or *worlds*) whenever a quantum measurement has $n$ possible outcomes, this is nevertheless the common way in which MWI is discussed, and we will to some extent use that language here. The other class contains all other interpretations of quantum mechanics. In these non-MWI interpretations, there is only ever one world, and the wavefunction collapses whenever a measurement is taken: if a measurement has $n$ possible outcomes, only one is ever realised.

The famously accurate predictions obtained by calculating solutions to the Schrödinger equation do not depend upon the interpretation of quantum mechanics. However it is desirable from a realist perspective to distinguish between these interpretations. Having divided them into two classes, it is natural to start by asking whether we can distinguish between the classes, either in real-world or thought experiments. In particular, if we can show that the class of non-MWI interpretations can be rejected, then we are left simply with the many-worlds interpretation.

One of the better-known thought experiments which claims to distinguish between these classes is a variant of the classic Schrödinger's cat experiment in which the cat, or rather a human in place of the cat, is the observer [Teg14, Teg98, Lew00]. The life of the observer depends upon the outcome of an automatic quantum measurement called a *trial*. Many trials occur, one after the other on a regular basis. As described in more detail in §2, this *Everett Box* is able to distinguish between MWI and non-MWI interpretations. The argument is a probabilistic one: though the observer might get lucky and survive a few trials, continued survival in a non-MWI universe is extremely unlikely. On the other hand, a version of the observer is guaranteed to survive with probability 1 in one branch of a MWI universe, guaranteeing the so-called *Quantum Immortality* of that observer. The former non-MWI option is rejected as being too unlikely. Thus only MWI is a valid interpretation.

We call the line of argument summarised above and presented in §2 *Everett's argument* and to its conclusion as *Everett's conclusion*, more in homage to the genesis of these ideas in Everett's seminal thesis [Eve57b] than because Everett himself clearly formulated them. Indeed, Everett never formally defined this experiment, but variants of it have been given independently by several authors [Squ86, Teg98]. It is not universally accepted. It has been attacked from various directions, not least in terms of its real-world applicability, for instance around the definition of death (or at least of a discrete binary distinction between "alive" and "dead") — see [Teg14, Teg98]. From the philosophical perspective we see critiques based on the classical philosophical problem of individual identity and its persistence, what it means to "expect" a subjective outcome like one's own death as opposed to predicting an objective event, the distinction between actual and probable events, and the meaning of probabilistic thinking in the MWI context; see [Lew00, Pap04, Ara12, Seb15, Vai18], and references therein. Everett himself anticipated some of these objections in [Eve57b].

There are two other ways in which Everett's argument is critiqued, both of which are much more general in their scope. They concern (1) the role of infinity and the infinitesimal in physics, and (2) the role of the computable. A motivation for the first of these is that if we live in a finite universe which has existed for finite time, and if the fields, matter, time, and space of the universe are all discrete at small enough scales discrete, then we should reject all objects and arguments which employ the infinite and the infinitesimal. Such strictly *finitist* theories include digital physics [Whe90], cellular automata [Wol02], loop quantum gravity [RS88], and more besides — see [Sch97] and references therein. The questionable role of infinity in Everett's argument has been highlighted by

[Teg14] amongst others.

The second critique, namely that our current theories of physics are non-computable, is the focus of the present work. Requiring a computable theory of physics is essentially the same as requiring all knowledge to be obtained through an algorithmic process in finite time. It is not the same as requiring only finite objects, but it does necessitate working within so-called non-classical logics, as we outline in detail below. The desirable quality of computability in the foundations of physics is not obtained by classical logic.

Thus while probabilities and probabilistic thinking have been highlighted as potential concerns with Everett's argument [Lew00, Pap04], and while the role of the infinite and the infinitesimal in physics [Teg14] have also been called into question in this context, to our knowledge no-one has examined the argument from a computable perspective before, and in particular from within non-classical logic.

Here we show that Everett's argument that we must reject all non-MWI interpretations of quantum mechanics is based on a classical understanding of limiting behaviours of functions which need not hold in other, non-classical logics. In particular, we show that the argument fails in a constructive logic called Recursive Constructive Mathematics, commonly referred to as RUSS, in which all results are computable. Within RUSS, we show that the existence of so-called *pathological* probability distributions mean that we must reject Everett's argument that all non-MWI interpretations are wrong. However, we are able to show through a new argument that Everett's conclusion holds even in a universe (or universes) governed by such non-classical logics, thus strengthening the argument in favour of the MWI and requiring any counter-arguments to also be valid in these non-classical logics.

In §2 we give a brief overview of Everett's argument for how the Everett Box implies a rejection of all non-MWI interpretations of quantum mechanics. Next, in §3 we define computability and outline the arguments in favour of requiring computability in our theories of physics, before giving a summary of the main result from [MJW19] on which we base the principal argument in this paper. With this background we prove in §4 the Pathological Immortality Theorem, which shows that Everett's argument does not work in RUSS. However, in §5 we present a constructive, computable proof that Everett's conclusion nevertheless holds in a universe whose logic is that of RUSS. Finally, we summarise and discuss our results in §6.

## 2 Quantum Immortality

A conscious observer is placed in a box with a lethal quantum apparatus. The contents of the box are completely isolated from the rest of the universe. Although the thought experiment does not depend on the details of the lethal apparatus, a particularly clear example is given by [Teg98] and called the "quantum gun". The quantum gun consists of a gun coupled to a quantum system of a particle in a superposition of two states. At regular time intervals, a measurement of this system is made automatically, and if it is found to be in one state the gun fires a bullet, while if it is in the other nothing happens. After either firing or not firing, the quantum gun resets: a new superposition is set up and the memoryless process repeats[1]. Each independent occurrence of this process we call a *trial*. We take this or a similar lethal setup to be indefinitely repeatable and to occur every second[2]. The apparatus and the observer are isolated from the rest of the universe, and this setup constitutes the Everett Box.

What is the experience of the observer? It is rather starkly illustrated by Tegmark's gun if we contrast the Everett Box with a similar experiment in which instead of being aimed at the observer the gun merely fires or does not fire depending on the measurement. In this case, the observer can expect to hear a random string of bangs and clicks: the bangs correspond to the gun firing, the clicks to it not firing and the equipment resetting. Over time the relative proportion of bangs and clicks will tend towards the relative likelihoods of those two outcomes. In the standard formulation, both outcomes occur with equal probability and thus the observer expects over time that 50% of the sounds will be bangs, and 50% clicks. Everett's argument is actually independent of these likelihoods, which need not be either equal or constant [Eve57b, Eve57a, Teg14]. It is such a general case we consider in this paper.

The preceding description is not that of the Everett Box, because the life and hence consciousness[3] of the observer does not depend upon the outcome of the measurement. In the Everett Box, the gun is aimed at the observer in such a way that should it fire then death is certain and swift[4]. In this case, what should the observer expect?

The answer depends upon which class of interpretation of quantum mechanics holds in our universe. If there is only one world, then each trial involves the collapse of the wavefunction and a single outcome occurs for the observer: either they hear a "click" or they are instantly killed (and so hear nothing). They might get lucky once, they might get lucky twice, but as time goes on and the number of trials increases, the odds of them surviving decreases exponentially.

If, however, there are many worlds, the totality of which contain all possible outcomes and histories, then by necessity there is always an observer alive after any number of trials. For example, after one trial there are two versions of the observer, the universe having branched into two worlds at the moment of the quantum measurement. In one world the observer heard "click" while in the other they died. After two seconds there are three worlds. In one of them, the observer's history shows that they heard "click-click". In another world, they heard "click" and then died on trial 2. In the third they died on trial 1. After three trials there is an observer whose history is "click-click-click", and after any number, $m$, of trials there will always be one world in which the observer has heard $m$ clicks. After a large number of trials there are many worlds, in all but one of which the observer is dead, but crucially there remains one living observer. Thus the subjective probability of surviving $m$ trials is 1 for any $m$, because there is a world in which the observer is still alive after any number of trials.

Thus from the observer's perspective this experiment has the potential to distinguish between the two classes of interpretation, though the stakes are high. The argument runs as follows. The chances of remaining alive after a large

[1] This slight variant of Tegmark's quantum gun of [Teg98] was given in [Teg14].

[2] The time interval is not important to the subsequent argument, other than to allow for many repetitions within a human lifetime.

[3] Consciousness surviving death is not a part of the thought experiment.

[4] Both conditions are necessary as outlined in [Teg98].

number of trials in a non-MWI universe is monotonically and exponentially decreasing because each trial is independent of the preceding trials. Thus at some point the probability of being alive will be lower than some threshold at which the still-alive observer can reject the non-MWI class of interpretations purely on the grounds of the low probability of such a sequence of events occurring. This is a standard experimental approach, and as usual the threshold $\epsilon \ll 1$ could be set to the traditional $5\sigma$-level, or indeed to a level of any stringency due to the monotonicity of the probability of remaining alive. Furthermore, with each subsequent survived trial the confidence in rejecting non-MWI interpretations increases. Of course, if we do not live in an MWI universe then the experiment simply kills the observer within a short time. They do not know that they do not live in a MWI universe, but neither do they know anything ever again.

In more rigorous terms, in non-MWI interpretations the probability of being dead after $m$ trials, $P(m)$ is, in the standard presentation in which the probability of death at each trial is 50%, simply

$$P(m) = 1 - \left(\frac{1}{2}\right)^m$$

which tends to unity as $m \to \infty$. The same conclusion holds regardless of the probability of staying alive on trial $k$, which we denote $p_k$. In this case, because $p_k < 1$ for all $k$ we still have

$$P(m) = 1 - \prod_{k=1}^{m} p_k \to 1 \quad \text{as} \quad m \to \infty. \tag{1}$$

While (1) will remain true throughout this paper, we will see that in the computable logic RUSS we can no longer use it to conclude that the observer necessarily must expect to be dead after any finite number of trials. First, we must review what it means to be computable.

# 3 Computability and The Infinite Monkey Theorem

## 3.1 Computability and Logic

A problem is said to be *computable* if it can be solved in an effective manner, which can be more formally defined in a number of models of computation [Coo04, CPS13, Bri94]. Loosely speaking, computable problems are those which can be solved algorithmically in finite time. The major milestone in computability theory is the Turing-Church thesis identifying computable functions on the natural numbers with functions computable on a Turing machine [Bri94, BP18, Dea20, CPS13].

It is not simply the rise in computer simulations, nor the "shut-up-and-calculate" instrumentalist approach to physics [Mer04], which have led some authors to suggest that computability should be a requirement for our theories of physics [Zus69, RS88, Sch97, tH99, Fre03, Llo05, Wol02]. It is instead the notion of the effective method embedded in computability that is important. A method is called effective for a class of problems when it comprises a finite set of instructions which can be followed by a mechanical device[5], that these instructions produce a correct answer, and that they finish after a finite number of steps [CPS13].

The desirability for computability in physics is therefore a product of a desire to know, and a belief that the universe is ultimately comprehensible to us. The reasoning in the syllogism goes that if we accept the two premises that (1) the universe is entirely comprehensible to the human mind, and (2) there is nothing extra-computational happening in the human mind, then we must accept the conclusion that physics is necessarily computable.

However, our current theories of physics are not computable, built as they are on classical mathematical ideas which in turn rely on classical, non-computable, logic [BP18]. There are two issues here. The first concerns the notion of infinity and the related notion of continuity. Infinities abound in our physical theories, whether they are in limiting behaviours (as examined in non-classical logics in the present paper) or in the related idea of continuous matter or continuous fields. In the latter case, even though we know that neither matter nor fields are continuous in our universe, we treat the "gap" between our continuous theories and discrete nature as being essentially a rounding error: the high accuracy of predictions made with the (presumptively Platonic) continuous theories is because our universe is approximately continuous. It is, after all, perhaps only discrete below the Planck length, or on time scales shorter than the Planck time.

The second issue, and the one that concerns us in this paper, is the notion of the underlying logic of the universe. Classical logic is not computable, relying as it does on non-computable notions such as the Law of Excluded Middle (LEM) and omniscience principles [BR87, BP18]. Why should we work with a logic that does not allow for computability if we wish our physics to be computable? One answer is similar to the response to continuity and infinity: because this logic works, to an astonishing degree [Wil18]. A second response is simply to reject the second premise given above. Perhaps there is something extra-computational happening within the human mind[6]. This is consistent with a robustly Platonic vision of the universe. If mathematical objects exist in a Platonic realm of forms to which our minds (somewhat mysteriously) have access, then the necessity of computability can be rejected. This is also consistent with the view above that our physical universe is only an (albeit excellent) approximation to a Platonic form.

If however we insist with the authors above that our logic must be computable, then we necessarily have to work with non-classical logics which are computable. In particular, we should work within so-called *constructive* interpretations of logic [BP18], in which the classical interpretations of disjunction and existence are rejected in favour of constructive ones. For example, the quantifier "there exists" becomes "we can construct (that is, give an effective method for defining) an object for which the given statement is true". There are several varieties of constructive mathematics [BR87]. It should be noted that not all varieties reject notions of infinity. Bishop's Constructive Mathematics (referred to as BISH) [BB85], for example, admits many classical mathematical objects which rely on infinities and continuity, but insists that proofs using these objects must proceed constructively (and are therefore computable). This illustrates the

[5]The idea here is not that they must be followed by such a device, but that even a human following them needs no *ingenuity* in order to derive a correct answer.

[6]This perspective overlaps somewhat with the notion of *hypercomputation* [Cop02, Cop04].

important distinction between the *epistemological constructivism* of BISH which remains agnostic on the ontology of mathematical objects, and the *ontological constructivism* of other varieties of constructive mathematics which insist that both objects and proofs (procedures) must be computable [BP18, BR87]. It has been said that computable mathematics is simply mathematics done with intuitionistic logic [BP18].

In order to subject Everett's argument to a strong scrutiny in a non-classical logic, we here choose an ontologically constructive variety of constructive logic, Recursive Constructive Mathematics, RUSS [BP18]. RUSS is a constructive version of recursive function theory, in which functions on the natural numbers are defined recursively. Essentially, RUSS takes the classical recursive analysis in the tradition of Turing and Church but uses only intuitionistic logic. In the following subsection, we briefly outline the theorems of a recent work in computable probability based on RUSS which will be central to the argument of this paper.

### 3.2 The Infinite Monkey Theorem

Working in RUSS, [MJW19] proved a seemingly counter-intuitive theorem, which we call here the *Infinite Monkey Theorem (IMT)*. To state the IMT we first need some notation. The IMT was written in the playful language of the famous aphorism that a large enough group of monkeys with typewriters will reproduce the complete works of Shakespeare, but as is made clear in [MJW19], the IMT is really about computable probability distributions, as indeed is our focus in the present paper.

Retaining the metaphor of [MJW19], we work in an alphabet $A$ (of size $|A|$, including punctuation) and call a *w-string* any string of characters of length $w \in \mathbb{N}$ . For example, "*banana*" is a 6-string over the alphabet $\{a, b, n\}$. Each monkey works on a computer keyboard with $|A|$ unique keys and each monkey types a $w$-string in finite time. We define $M$ to be an infinite, enumerable set of monkeys (the *monkeyverse*), and for any $m \in \mathbb{N}$ the *m-troop* of monkeys to be the first $m$ monkeys in $M$. We then have

**Theorem 1** (Infinite Monkey Theorem)**.** *Given a finite target $w$-string $T_w$ and a positive real number $\epsilon$, there exists a computable probability distribution on $M$ of producing $w$-strings such that:*

*(i) the classical probability that no monkey in $M$ produces $T_w$ is* 0*; and*

*(ii) the probability of a monkey in* any *$m$-troop producing $T_w$ is less than $\epsilon$.*

[MJW19] established an even stronger, target-free version of this theorem, which requires only a knowledge of $w$, not of $T_w$.

The theorem and its proof are computable. That is, while it is classically true that it is impossible that no monkey reproduces the works of Shakespeare (part (i)), it is possible to construct a so-called *pathological* probability distribution on the monkeyverse such that the chances of actually finding the monkey that does so can be made arbitrarily small (part (ii)). The key point in part (ii) is that this is true for *any* finite $m$-troop of monkeys; the pathological distribution does not require knowledge of the size of the $m$-troop, it is simply pathological for all finite sets.

The monkeys correspond to any finite back-box process occurring in finite time. The general conclusion drawn in [MJW19] is that in a computable universe the space of all possible probability distributions on enumerable sets contains a non-empty set of pathological distributions for which the IMT holds. This is in contradistinction to a universe governed by classical logic in which the IMT does not hold. It is this distinction that we exploit in the remainder of the paper, by examining the impact of the existence of pathological distributions on the enumerable set of trials in the Everett Box.

# 4 Pathological Distributions Imply the Rejection of Everett's Conclusion

With the notation from §2 we can state that the probability $P(m)$ of dying within $m$ trials is given by

$$P(m) = 1 - \prod_{k=1}^{m} p_k \tag{2}$$

for any $m \in \mathbb{N}$, where $p_k$ is the probability of not dying on trial $k$. The key thing here, in contrast to the manner in which the Everett Box is normally described, but in keeping with the more general case which Everett himself allowed for in [Eve57b], we consider a quantum apparatus which gives varying probabilities at each trial. We note that it is not that the probability of death at each trial, $p_k$, cannot be known in advance; after all, in the standard formulation, $p_k = 0.5$ for all $k$. The restriction is that the observer cannot predict in advance whether she lives or dies on trial $k$, and that her fate is determined purely by the unknowable quantum state of the apparatus. We can now state the following theorem.

**Theorem 2** (Pathological Immortality Theorem)**.** *While classically it is impossible that an observer in an Everett box remains alive as the number of trials tends to infinity, there is a computable probability distribution on the trials such that the probability that the observer is alive after* any *finite number of trials is arbitrarily close to 1.*

*Proof.* The result follows directly from the proof of the IMT in [MJW19]. In particular, we place the objects of the IMT and the objects of the Pathological Immortality Theorem (PIT) in one-to-one correspondence as outlined in the following table.

| | IMT | PIT |
|---|---|---|
| $p_k$ | probability that $k^{\text{th}}$ monkey fails to reproduce Shakespeare | probability of not dying on $k^{\text{th}}$ trial |
| $P(m)$ | probability that $m$-troop does reproduce Shakespeare | probability of dying within $m$ trials |

□

The PIT therefore gives us the apparently counterintuitive result that while classically it remains true that the observer's probability of being alive after $m$ trials tends to 0 as $m$ tends to infinity, the classical interpretation of that result as being that after a certain *finite* number of trials the probability of the observer being alive should be so small that she should be surprised at remaining alive is not true in a computational sense, in which that probability can remain arbitrarily close to 1 for *any* finite number of trials. As outlined in [MJW19], it is important to note that the apparent contradiction here is only between the classical notion of the limit and the existence of computable pathological distributions within RUSS; we are deliberately comparing results from non-commensurate logical systems in order to show that classical logic may lead us astray in a computable universe.

At first blush the PIT would seem to suggest that in a universe run on computable logic[7] Everett's conclusion is no longer valid. If the quantum apparatus happens to produce a pathological distribution then it is no longer unlikely that the observer remains alive after any finite number of trials, since that likelihood can remain arbitrarily close to 1. As a result, since the observer can never know for sure that she is not in a pathological distribution, she cannot surely state that remaining alive after any finite number of trials is unlikely and so she can never reject non-MWI interpretations of quantum mechanics. However, in the next section we argue that Everett's conclusion should still be taken to hold even in a computable universe and even with the existence of the PIT.

# 5 Everett's Conclusion Restored

We state our main result as a theorem.

**Theorem 3** (Computable Everett Box)**.** *The Everett Box implies the rejection of non-MWI interpretations of quantum mechanics even in a computable universe modelled by RUSS.*

*Proof.* Suppose an observer in an Everett Box in a RUSS-universe has survived many trials. There are two situations to must consider depending on whether the probability distribution on the trials is pathological or not. To reiterate, the observer does not know and has no way of knowing which situation holds.

First, if the probability distribution is not pathological then the standard, classical-logic Everett argument holds, and the observer concludes that she must reject all non-MWI interpretations of quantum mechanics.

On the other hand, suppose that the distribution is pathological. Since Theorem 5 of [MJW19] showed that such distributions are vanishingly rare then the observer must reject all non-MWI interpretations since in a single world the odds of being in such a distribution are vanishingly small. However, in the MWI there will always be an alive variation of the observer in a pathological distribution and hence being one such consciousness is not surprising. □

To state the proof in other terms, we note that in a classical universe, so Everett's original argument goes, the observer rejects non-MWI interpretations because the odds of surviving repeated trials are so low, whereas in a RUSS computable universe she reject non-MWI interpretations for the same reason if she happens to be in a non-pathological distribution, or because the odds of being in a pathological situation where the PIT holds in a single world are

[7] We take this to be equivalent to the requirement for computability in the logic we use in our theories of physics.

also vanishingly low. The observer does not need, therefore, any knowledge of whether she is in such a pathological experiment since either way she must reject non-MWI interpretations on the same basis: namely, the unlikelihood of being in that situation if there is only one world.

# 6 Discussion

We have argued that Everett's thought experiment implies the rejection of all non-MWI interpretations of quantum mechanics even when the computability requirement is added to physics through employing RUSS, a constructive, computable logic. Our main point is therefore that those whose rejection of Everett's conclusions depends on some future recasting of physics in a computable form do not have that option if RUSS is the correct logic on which to base physics. We have shown, in fact, that Everett's argument still holds in at least one computable logic. The case to be made against Everett's conclusion therefore must be stronger than has previously been appreciated.

This naturally raises the question as to whether Everett's conclusion holds in other computable logics. For example, can we reproduce the argument in BISH, a computable logic which preserves most classical mathematical objects, including some of those which involve either infinity or continuity? What about in other logics which do not allow for such objects? And of course, what happens in a completely finitist universe?

We have three final points to make. The first is to point out that although the argument here is given in favour of Everett's conclusion, the argument behind the proof of PIT works in any situation, quantum or otherwise, in which the probability of an event occurring tending to 1 in the limit of an infinite sequence of trials is taken to mean that the probability of that event not having happened in any finite sequence of trials necessarily tends to 0. In RUSS, this is not true.

The second point is to contrast the number of ways in which quantum immortality is possible in the classical world and in RUSS. In the former, there is always a single branch of the universe which contains a living consciousness regardless of the number of trials. But in RUSS, there are many such worlds: both the classical survivor and any observer who finds herself in a pathological distribution. There are in fact countably infinitely many of these even though they are vanishingly rare, since the requirement for a distribution to be pathological is simply that $p_k > 1 - \epsilon$ for all $k$ [MJW19]. This is similar to the natural numbers having measure zero in the reals.

The final point is to observe that one can consistently reject Everett's argument without rejecting the MWI. There are compelling reasons for adopting the MWI as a formal mathematical approach to quantum mechanics, separate from any metaphysical issues of what "worlds" mean, how probabilities are to be interpreted when all possibilities are actually realised in the set of all worlds, and how that affects one's decisions — see [Deu99, Vai18] and references therein. Nevertheless our central idea remains that if one accepts Everett's argument in a theory built on classical logic, then one must accept it even in a computable theory built on RUSS, and that arguments against it must also work or have their counterparts in a computable theory.

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